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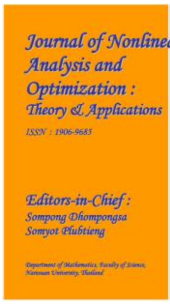

A GLIMPSE OF GAME THEORY

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Abstract

Game theory is a theoretical framework for conceiving social situations among competing players. In some respects, game theory is the science of strategy, or at least the optimal decision-making of independent and competing actors in a strategicsetting.

Key words Strategy, Player, Zero sum Games,

Introduction

Game theory is the study of mathematical models of strategic interactions among rational agents. It has applications in all fields of social science, as well as in logic, systems science and computer science.

The concepts of game theory are used extensively in economics as well .The traditional methods of game theory addressed two-person zero-sum games, in which each participant's gains or losses are exactly balanced by the losses and gains of other participants.

In the 21st century, the advanced game theories apply to a wider range of behavioral relations; it is now an umbrella term for the science of logical decision making in humans, animals, as well as computers.

GAME THEORY-WHAT IS IT?

The concepts of game theory provide a common language to formulate structure, analyse and eventually understand different strategical scenarios. Generally, game theory investigates conflict situations, the interaction between the agents and theirdecisions.

A game in the sense of game theory is given by a (mostly finite) number of players, who interact according to given rules. Those players might be individuals, groups, companies, associations and so on

Their interactions will have an impact on each of the players and on the whole group of players, i.e. they are interdependent.

To be more precise: A game is described by a set of players and their possibilities to play the game according to the rules, i.e. their set of strategies.

This description leads to a widespread definition of game theory

The subject of game theory is situations, where the result for a player does not only depend on his own decisions, but also on the behavior of the other players.

Game theory has its historical origin in 1928. By analyzing parlor games, Johnvon Neumann realized very quickly the practicability of his approaches for the analysisof economic problems.

Game Theory – Where is it applied?

As we have seen in the previous section, game theory is a branch of mathematics. Mathematics provide a common language to describe these games. We have also seen that game theory was already applied to economics by von Neumann.

When there is competition for a resource to be analyzed, game theory can be used either to explain existing behavior or to improve strategies.

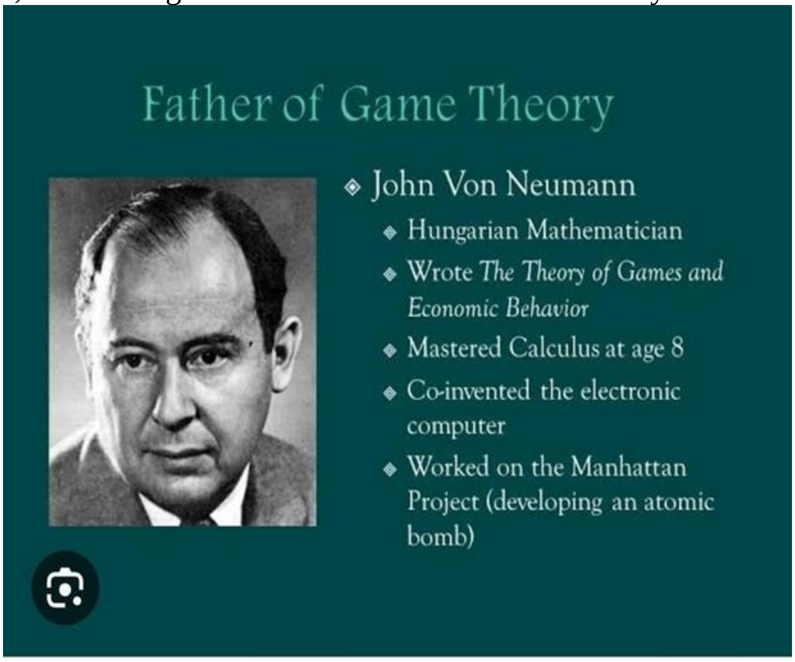
The first is especially applied by sciences which analyze long-term situations, like biology or sociology. In animality , for example, one can find situations, where cooperation has developed for the sake of mutual benefits.

The latter is a crucial tool in sciences like economics. Companies use game theory to improve their strategically information.

Despite the deep insights he gained from game theory's applications to economics, von Neumann was mostly interested in applying his methods to politics andwarfare, perhaps descending from his favorite childhood game, Kriegspiel, a chess-likemilitary simulation.

History of Game Theory

The field of game theory gained significant recognition with the publication of "Theory of games and economics behavior" by john von Neumann and Oscar Morgenstern in 1944.
The individual closely associated with the creation of the theory of games is
John von Neumann, one of the greatest mathematicians of this century.



Although others preceded him in formulating a theory of games - notably Emile Borel - it was von Neumann who published in 1928 the paper that laid the foundation for the theory of two-person zero-sum games.

The theory of Games was born in 1944 with the publication of Theory of Games and Economic Behavior by Hungarian-born American mathematician **John von Neumann** and his Princeton University colleague **Oskar Morgenstern**, a German-born American economist. In their book, They observed that economics is much like agame, wherein players anticipate each other's moves, and therefore requires a new kindof mathematics, which they called game theory.

Their choice of title was a little unfortunate, since it quickly got shortened to"Game Theory,"

COOPERATIVE / NON COOPERATIVE

A game is cooperative if the players are able to form binding commitments externally enforced (e.g. through contract law).

A game is non-cooperative if players cannot form alliances or if all agreements need to be self-enforcing (e.g. through credible threats).

Cooperative games are often analyzed through the framework of cooperativeDifferent types of games, which focuses on predicting which coalitions will form, thejoint actions that groups take, and the resulting collective payoffs. It is opposed to the traditional non-cooperative game theory which focuses on predicting individual players' actions and payoffs and analyzing Nash equilibria.

As non cooperative game theory is more general, cooperative games can be analyzed through the approach of non-cooperative game theory (the converse does not hold) provided that sufficient assumptions are made to encompass all the possible strategies available to players due to the possibility of external enforcement of cooperation.

CLASSIFICATION OF GAME THEORY

It broadly classified into three main sub-categories of study

1. Classical game theory

It focuses on optimal play in situations where one or more people must make a decision and the impact of that decision and the decisions of those involved is known. Decisions may be made by use of a randomizing device like piping a coin .It focuses on questions like, What is my best decision in a given economic scenario, where a reward function provides a way for me to understand how my decision will impact my result.

Examples: Poker, Strategic military decision making, Negotiation.

2. Combinatorial game theory

It focuses on optimal play in two-player games in which each player takesturns changing in pre-defined ways. In other word, combinatorial game theory does notconsider games with chance (no randomness)

Generally two player strategic games played on boards. Moves change thestructure of a game board.

Examples: Chess, Checkers, Go.

3. Dynamic game theory:

It focuses on the analysis of games in which players must make decisionsover time and in which those decisions will affect the outcome at the next moment in time.

It often relies on differential equations to model the behaviour of players over time.

It can help optimize the behavior of unmanned vehicles or it can help youcapture your baby sister who has escaped from her playpen.

In general games with time, Games with motion or a dynamic component.

Examples: Optimal play in a dog fight, chasing your brother across a room.

Nash equilibrium

If each player has chosen a strategy and no player can benefit by changing hisstrategy while the other players keep their unchanged, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium. John Nashshowed in 1950, that every game with a finite number of players and finite number of strategies has at least one mixed strategy Nash equilibrium.

BEST RESPONSE

The best response is the strategy (or strategies) which produce the most favorable immediate outcome for the current player, taking other players' strategies asgiven.

With this definition, we can now determine the Nash equilibrium in a normalform game very easily by using the payoff matrix.

LOCALIZING A NASH EQUILIBRIUM IN A PAYOFF-MATRIX

Let us use the payoff matrix of the Prisoner's Dilemma

Player1\Player2	C	D
C	3, 3	0, 5
D	5, 0	1, 1

The procedure is the following:

First we consider the options for player 1 by a given strategy of player 2, i.e.we look for the best answer to a given strategy of player 2

If player 2 plays C, the payoff for player 1 for choosing C will be 3, for choosingD it will be 5, so we highlight his best answer, D:

	C	D
C	3,3	0,5
D	5,0	1,1

The Nash equilibrium is then determined by the matrix element, in which both players marked their best answers.

Thus, the strategies that constitute a Nash equilibrium is defection by both players,because if any player changed his strategy to C whereas the other one stays with D, hewould get a less payoff.

NORMAL FORM GAMES

1. A game in normal form consists of:
2. A finite number of players.
3. A strategy set assigned to each player. (e.g. in the Prisoner's Dilemma eachplayer has the possibility to cooperate (C) and to defect (D). Thus his strategy set consists of the elements C and D).
4. A payoff function, which assigns a certain payoff to each player depending on his strategy and the strategy of the other players (e.g. in the Prisoner's Dilemma the time each of the players has to spend in prison.

EXAMPLE:

Player 1/player 2	L	R
U	1,3	2,4
D	1,0	3,3

In this example, player 1 (vertical) has two different strategies: Up (U) and Down(D). Player 2 (horizontal) also has two different strategies, namely Left (L) and Right (R).

The elements of the matrix are the outcomes for the two players for playing certainstrategies, i.e. supposing, player 1 chooses strategy U and player 2 chooses strategy R,the outcome is (2, 4), i.e. the payoff for player Is 2 and for player 2 is 4.

Contrary to the normal form game, the rules of an extensive form game are described such that the agents of the game execute their moves consecutively.

This game is represented by a game tree, where each node represents every possible stage of the game as it is played. There is a unique node called the initial nodethat represents the start of the game.

Any node that has only one edge connected to it is a terminal node and represents the end of the game (and also a strategy profile). Every non-terminal node belongs to a player in the sense that it represents a stage in the game in which it is thatplayer's move.

Every edge represents a possible action that can be taken by a player. Every terminal node has a payoff for every player associated with it. These are the payoffs forevery player if the combination of actions required to reach that terminal node are actually played.

EXAMPLE:

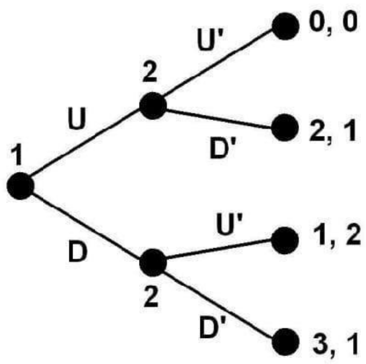


Figure 1: A game in extensive form.

In figure 1 the payoff for player 1 will be 2 and for player 2 will be 1, provided that player 1 plays strategy U and player 2 plays strategy D'.

Definition: The strategic form, or normal form, of a two-person zero-sum game isgiven by a triplet (X, Y, L), where

(1) X is a nonempty set, the set of strategies of Player I

(2) Y is a nonempty set, the set of strategies of Player II

(3) L is a real-valued function defined on X × Y. (Thus, L(x, y) is a real numberfor every x ∈ X and every y ∈ Y.)

The interpretation is as follows. Simultaneously, Player I chooses x ∈ X and Player II chooses y ∈ Y, each unaware of the choice of the other. Then their choices aremade known and I wins the amount L(x, y) from II. Depending on the monetary unit involved, L(x, y) will be cents, dollars, pesos, beads, etc. If L is negative, I pays the absolute value of this amount to II. Thus, L(x, y) represents the winnings of I and the losses of II.

Constitutes one strategy. The program Deep Blue, that beat world chess champion Gary Kasparov in a match in 1997, represents one strategy. The set of all such strategies forPlayer I is denoted by X. Naturally, in the game of chess it is physically impossible to describe all possible strategies since there are too many; in fact, there are more strategies than there are atoms in the known universe. On the other hand, the number of games of tic-tac-toe is rather small, so that it is possible to study all strategies and find an optimal strategy for each player. Later, when we study the extensive form of a game, we will see that many other types of games may be modeled and described in strategic form.

To illustrate the notions involved in games, let us consider the simplest non-trivial case when both X and Y consists of two elements. As an example, take the gamecalled Odd-or-Even.

Example: Odd or Even. Players I and II simultaneously call out one of the numbers one or two. Player I's name is Odd; he wins if the sum of the numbers if odd.Player II's name is Even; she wins if the sum of the numbers is even. The amount paidto the winner by the loser is always the sum of the numbers in dollars. To put this gamein strategic form we must specify X , Y and L. Here we may choose X = {1, 2}, Y = {1, 2}, and L as given in the following table.

		II (even) y	
		1	2
	1	-2	+3
I (odd) x	2	+3	-4

L(x, y) = I's winnings = II's losses.

It turns out that one of the players has a distinct advantage in this game.

Let us analyze this game from Player I's point of view. Suppose he calls 'one'3/5ths of the time and 'two' 2/5ths of the time at random. In this case,

1. If II calls 'one', I loses 2 dollars 3/5ths of the time and wins 3 dollars 2/5ths of thetime; on the average, he wins −2(3/5) + 3(2/5) = 0 (he breaks even in the long run).
2. If II call 'two', I wins 3 dollars 3/5ths of the time and loses 4 dollars 2/5ths of thetime; on the average he wins 3(3/5) − 4(2/5) = 1/5.

That is, if I mixes his choices in the given way, the game is even every time II calls 'one', but I wins 20/c on the average every time II calls 'two'. By employing thissimple strategy, I is assured of at least breaking even on the average no matter what II does. Can Player I fix it so that he wins a positive amount no matter what II calls?

Let p denote the proportion of times that Player I calls 'one'. Let us try to choosep so that Player I win the same amount on the average whether II calls 'one' or 'two'.Then since I's average winnings when II calls 'one' is −2p + 3(1 − p), and his averagewinnings when II calls 'two' is 3p − 4(1 − p) Player I should choose p so that

$$-2p + 3(1 - p) = 3p - 4(1- p)$$

$$3 - 5p = 7p - 4$$

$$12p = 7$$

$$p = 7/12.$$

Hence, I should call 'one' with probability 7/12, and 'two' with probability 5/12. On the average, I wins −2 (7/12) + 3(5/12) = 1/12, or 8 (1/ 3) cents every time heplays the game, no matter what II does. Such a strategy that produces the same averagewinnings no matter what the opponent does is called an equalizing strategy.

Therefore, the game is clearly in I's favor. Can he do better than 8 (1/ 3) centsper game on the average? The answer is: Not if II plays properly. In fact, II could use the same procedure:
call 'one' with probability 7/12call 'two' with probability 5/12.

If I calls 'one', II's average loss is −2 (7/12) + 3(5/12) = 1/12. If I calls 'two',II's average loss is 3(7/12) − 4(5/12) = 1/12.

Hence, I have a procedure that guarantees him at least 1/12 on the average, andII has a procedure that keeps her average loss to at most 1/12. 1/12 is called the value of the game, and the procedure each uses to insure this return is called an optimal strategy or a minimax strategy.

If instead of playing the game, the players agree to call in an arbitrator to settlethis conflict, it seems reasonable that the arbitrator should require II to pay 8 (1/ 3) centsto I. For I could argue that he should receive at least 8 1/ 3 cents since his optimal strategy guarantees him that much on the average no matter what II does. On the otherhand II could argue that he should not have to pay more than 8 1/ 3 cents since she hasa strategy that keeps her average loss to at most that amount no matter what I do.

THE MINIMAX THEOREM

A two-person zero-sum game (X, Y, L) is said to be a finite game if both strategy sets X and Y are finite sets. The fundamental theorem of game theory due to von Neumann states that the situation encountered in the game of Odd-or-Even holds for all finite two-person zero-sum games. Specifically, The Minimax Theorem. For every finite two-person zero-sum game,

(1) There is a number V , called the value of the game,

(2) There is a mixed strategy for Player I such that I's average gain is at least V no matter what II does, and

(3) There is a mixed strategy for Player II such that II's average loss is at most V no matter what I do.

This is one form of the minimax theorem to be stated more precisely and discussed in greater depth later. If V is zero we say the game is fair. If V is positive, wesay the game favors Player I, while if V is negative, we say the game favors Player II.

Advertising War Coke vs. Pepsi:

Without any advertising, each company earns \$5b/year from Cola consumers.Each company can choose to spend \$2b/year on advertising. Advertising does not increase total sales for Cola, but if one company advertises while the other does not, itcaptures \$3b from the competitor

pepsi

No ad ad

No

adCoked

\$5b,\$5b	\$2b,\$6b
\$6b,\$2b	\$3b,\$3b

In your everyday life:

Everything is a game, poker, chess, soccer, driving, dating, stock market advertising, setting prices, entering new markets, building a reputation bargaining, partnerships, job market search and screening designing contracts, auctions, insurance, environmental regulations international relations, trade agreements, electoral campaigns, Most modern economic research includes game theoretical elements.

Definition:

A population state $x \in \Delta$ lists the prevalence of each of the pure strategiesin the population.

Remark. There are two possible interpretations for this:

1. Every agent is genetically hardwired to play the mixed strategy x.

2. Every agent is genetically hardwired to play a pure strategy: for each$i \in \{1, . . . , m\}$, x_i is the proportion of agents following the ith pure strategy, e^i.

Given that agents meet randomly in pairs, and the population is sufficiently large,the probability that a specific agent's randomly selected opponent plays the ith pure strategy can be taken to be x_i^3. Thus an agent following the strategy $y \in \Delta$ when the population state is $x \in \Delta$ will receive an expected payoff of u (y, x), while the averagepayoff in the population is u (x, x). Logically an agent following a strategy that does better than average will grow quicker and proportionately expand, shifting the population state.

Two person zero sum game (with more than **two players)**

TPZS games with **more than two people involved** are

(i) Team sports with only two sides, but with more than one player in each side

(ii) Many people involved in **surrogates for military conflict**, so it shouldcome as no surprise that many military problems can also be analyzed as TPZS games**.**

Pay Off

The outcome of the game resulting from a particular decision (or strategy) iscalled pay off. It is assumed that pay off is also known to the player in advance.

It is expressed in times of numerical values such as money, percent of marketshare or utility.

SOCIAL MEDIA AND PRIVACY

Game theory can be applied to model interactions between social media platforms, users, and advertisers to develop privacy – preserving mechanisms and dataprotection policies.

OVERVIEW OF GAME THEORY

Game theory is an approach to modeling behavior in situations where the outcome of your decisions depends on the decisions of others.

Game theory is the study of strategic, interactive decision making among rational individuals or organizations.

Game theory is a branch of applied mathematics that provides tools for analyzing situations in which parties (called players) make decisions that are inter dependent. This interdependence causes each player to consider the other player'spossible decisions (or strategies) in formulating stratergy.

In addition, a player need not be an individual; it may be a nation, a corporation, or a team comprising many people with shared interests.

A solution to a game describes the optimal decisions of the players, who may have similar, opposed, or mixed interests, and the outcomes that may result from these decisions. Game theory is applied for determining different strategies in the business world. It offers valuable tools for solving strategy problems.

Conclusion

It is evident from this paper that game theory goes beyond mathematical representations to the description of the real world events where decisions made by other players affect other players' interests. Game theory may not be very efficient in the prediction of behavior like in the case of sciences though in some unique cases, it might take up that role as described (Vorob'ev 1994). Game theory in this essay is looked at as a conceptual analysis applicable in the decision-making process as well asconflict resolution.

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